Monday Night Calculus, September 27, 2021

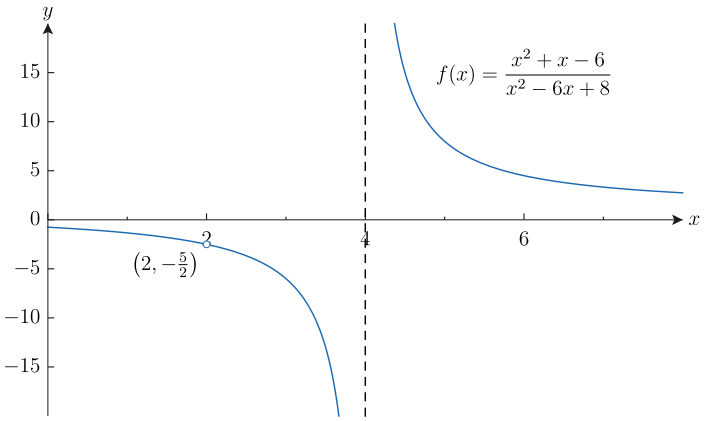
1. Limits involving $\dfrac{x^2+x-6}{x^2-6x+8}$ (Alexandra Pepin)

$$\lim_{x\to 2}\frac{x^2+x-6}{x^2-6x+8}=\lim_{x\to 2}\frac{(x+3)(x-2)}{(x-4)(x-2)}=\lim_{x\to 2}\frac{x+3}{x-4}=\frac{2+3}{2-4}=-\frac{5}{2}$$

$$\lim_{x\to 4^+}\frac{x^2+x-6}{x^2-6x+8}=\lim_{x\to 4^+}\frac{x+3}{x-4}=\frac{7}{(+)}=\infty$$

$$\lim_{x\to 4^-}\frac{x^2+x-6}{x^2-6x+8}=\lim_{x\to 4^-}\frac{x+3}{x-4}=\frac{7}{(-)}=-\infty$$

Some graphical evidence:



2. $\lim_{h\to 0} \dfrac{\sin\left(\frac{\pi}{6}+h\right)-\sin\left(\frac{\pi}{6}\right)}{h}$ (Andi Cadden Broussard via Virge's Limits Circuit)

$$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

$$\left.\frac{d}{dx}\sin x\right|_{x=\pi/6}=\lim_{h\to 0}\frac{\sin\left(\frac{\pi}{6}+h\right)-\sin\left(\frac{\pi}{6}\right)}{h}=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$$

$$\lim_{h\to 0}\frac{\sin\frac{\pi}{6}\cos h+\cos\frac{\pi}{6}\sin h-\sin\frac{\pi}{6}}{h}$$

$$=\lim_{h\to 0}\frac{\sin\frac{\pi}{6}(\cos h-1)+\cos\frac{\pi}{6}\sin h}{h}$$

$$=\frac{1}{2}\lim_{h\to 0}\frac{\cos h-1}{h}+\cos\frac{\pi}{6}\lim_{h\to 0}\frac{\sin h}{h}$$

3. The graph of $y = \dfrac{mx^3 + x + a}{x^3 - 2}$ crosses its horizontal asymptote at the point $(6, -5)$. What is the value of $m + a$? (Zach Sarver)

HA: $y = -5$

$$\lim_{x\to\infty} \frac{mx^3 + x + a}{x^3 - 2} = \lim_{x\to\infty} \frac{m + \dfrac{1}{x^2} + \dfrac{a}{x^3}}{1 - \dfrac{2}{x^3}} = m \quad \Rightarrow \quad m = -5$$

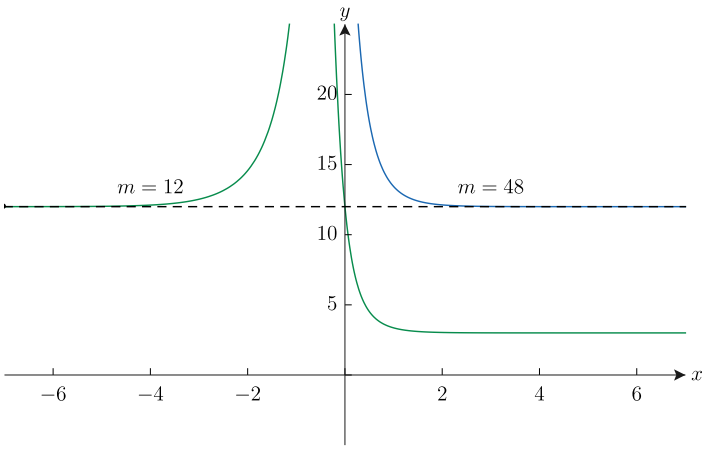
$$x = 6 \quad \Rightarrow \quad \frac{-5 \cdot 6^3 + 6 + a}{6^3 - 2} = \frac{-1074 + a}{214} = -5 \quad \Rightarrow \quad -1074 + a = -1070 \quad \Rightarrow \quad a = 4$$

$$m + a = -5 + 4 = -1$$

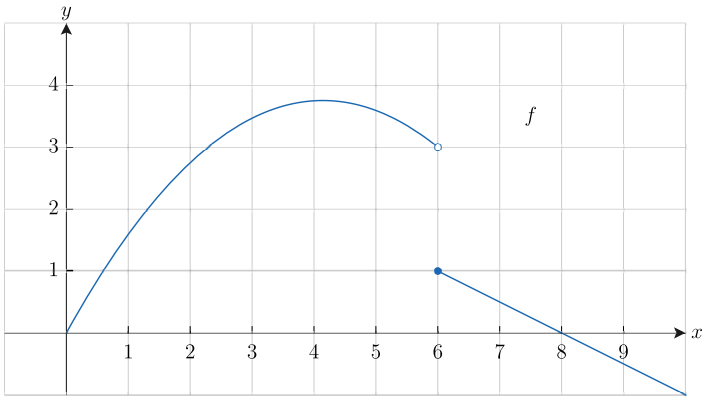
For which values of m will the line $y = 12$ be a horizontal asymptote on the graph of $y = \dfrac{mx + m \cdot 6^{-x}}{4x + 6^{-x}}$?

$$\lim_{x\to\infty} \frac{mx + m \cdot 6^{-x}}{4x + 6^{-x}} = \frac{m}{4} = 12 \quad \Rightarrow \quad m = 48$$

$$\lim_{x\to-\infty} \frac{mx + m \cdot 6^{-x}}{4x + 6^{-x}} = m = 12$$



4. The graph of the function f is shown in the figure. (Allison Myers Abell)



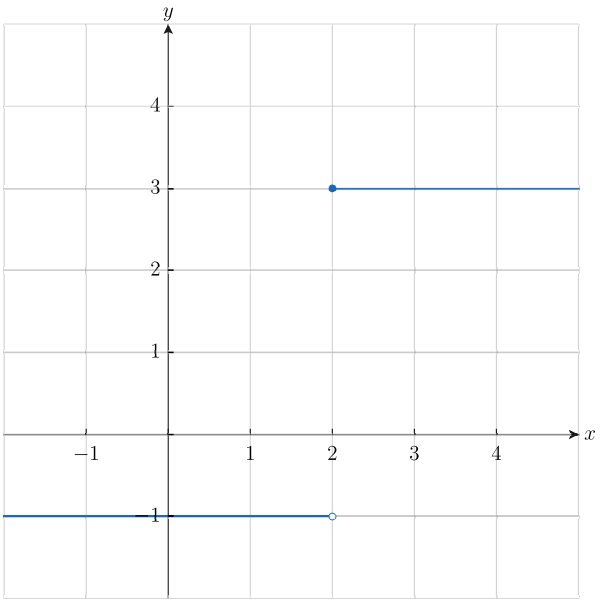
Find each limit, if it exists.

$$\lim_{h \to 0^+} \frac{f(6+h) - f(6)}{h} =$$

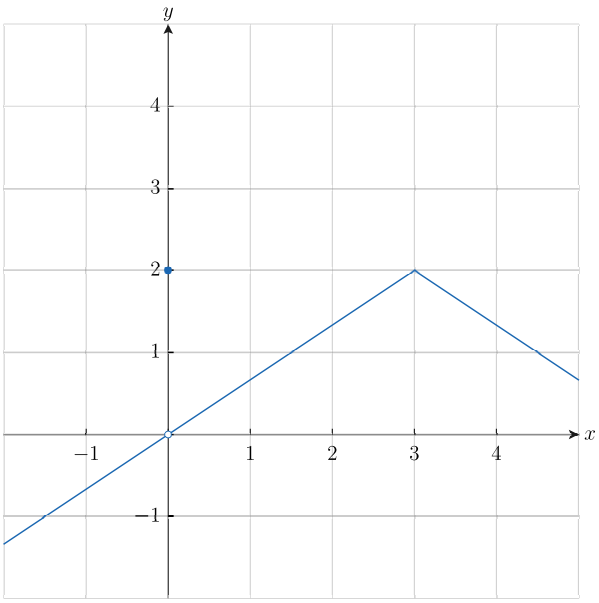
$$\lim_{h \to 0^-} \frac{f(6+h) - f(6)}{h} =$$

$$\lim_{h \to 0} \frac{f(6+h) - f(6)}{h} =$$

5. The graphs of the functions f and g are shown below. (Carla Duke Rossi)



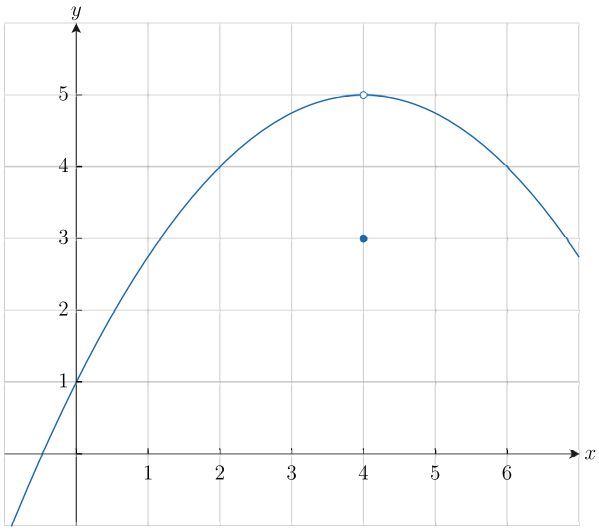
Graph of $y = f(x)$



Graph of $y = g(x)$

Find $\lim_{x \to 0} g(f(x) + 1)$

6. The graph of the function f is shown below. (Katherine Maxwell)



Find the value of $\lim_{x \to 4} 2\cos(f(x))$