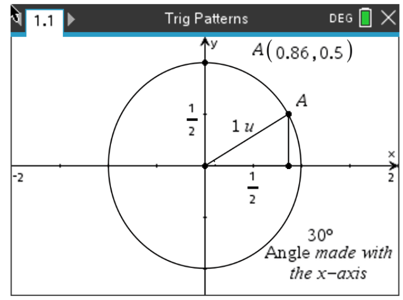
Trig Patterns

Math Objectives

- Students will use the unit circle and the handheld to estimate the six trig functions at certain angle measures.
- Students will then find patterns in the results of the six trig function estimations and discuss them with their classmates.
- Students will try to make a connection with how to understand these topics in IB Mathematics courses and on their final assessments.



Vocabulary

- Unit Circle
- Right Triangle Trigonometry
- Radian Measure

About the Lesson

- This lesson is aligning with the curriculum of IB Mathematics Applications and Interpretations HL and IB Mathematics Approaches and Analysis SL/HL
- This falls under the IB Mathematics Content Topic 3 Geometry and Trigonometry:

  AI HL 3.7: **(a)** The definition of a radian and conversion between degree and radian

  3.8: **(a)** The definition of $\cos\theta$ and $\sin\theta$ in terms of the unit circle

  AA SL/HL 3.4: The circle: radian measures of angles

  3.5: **(a)** Definition of $\sin\theta$, $\cos\theta$ in terms of the unit circle

  (b) Definition of $\tan\theta$ as $\frac{\sin\theta}{\cos\theta}$

  (c) Exact values of trig ratios of $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ and their multiples

  3.9: **(a)** Definition of the reciprocal trig ratios $\sec\theta$, $\csc\theta$, *and* $\cot\theta$

  As a result, students will:

  - Apply this information to real world situations.

TI-Nspire™ Navigator™

- Transfer a File.
- Use Class Capture to examine patterns that emerge.
- Use Live Presenter to demonstrate.
- Use Teacher Edition computer software to review student documents.
- Use Quick Poll to assess students' understanding

Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX II handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

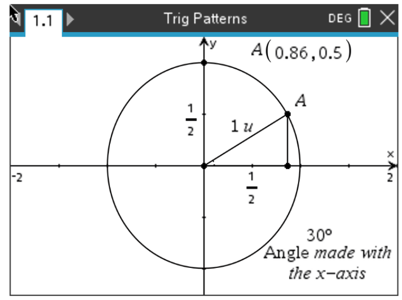
Lesson Files:

Student Activity
Nspire-TrigPatterns-Student.pdf
Nspire-TrigPatterns-Student.doc
Trig Patterns.tns

Activity Materials

Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

In this activity, students will use the unit circle to examine patterns in the six trigonometric functions. With the aid of the handheld and the file *Trig Patterns.tns*, students will compare angles created with the x-axis in all four quadrants and discuss with one another what is happening at each coordinate as they move the point around the circle.

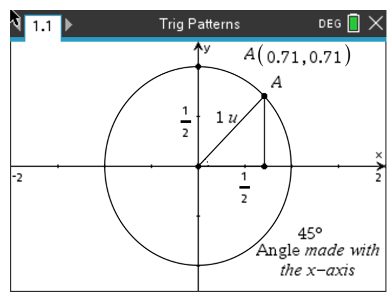


Problem 1 – Searching for Patterns

Using the unit circle, the trig functions can be defined as follows:

$$\sin\theta = \frac{y}{h} \qquad \cos\theta = \frac{x}{h} \qquad \tan\theta = \frac{y}{x}$$

Using the *Trig Patterns.tns* file, grab point A on the unit circle in the first quadrant by pressing and holding down on the center of the touch pad or by pressing **ctrl** then the center of the touchpad. Record the value for $\sin\theta, \cos\theta$ and $\tan\theta$ using the displayed x- and y-values, and the equations above.

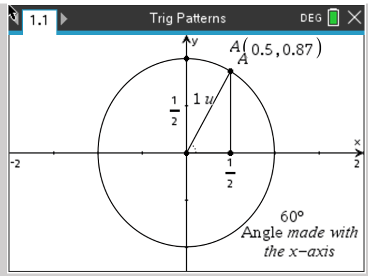


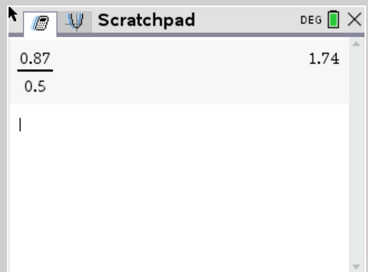
Use the radian conversion to fill in the second column: $\theta \cdot \frac{\pi}{180°}$

Teacher Tip: The angle measures will vary from the exact values in the answer tables due to the tns file only being able to measure coordinates to two decimal places. Make students aware that they may need to use the closest angle measure possible for some values.

Tech Tip: The tns file "*Trig Patterns.tns*", must be loaded on all calculators before the start of the activity.

Teacher Guidance: Students will move the triangle of the unit circle to find the angle measures listed in the table on the student worksheet. Students will record the values and then answer questions about the patterns in the results. Because the tns file only measures the angle less than 90°, there is opportunity for some further student learning. This means that when the angle being displayed is 30° and the point is in the second quadrant, the angle being observed is really 150° (180° - 30°). This can lead to discussions on reference angles. Students can write the ratios on their worksheets and then use a Calculator page or the Scratchpad to do their calculations.





Problem 1 – Complete the Table

θ	$Radian\ Measure$	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	**0**	**0**	**1**	**0**
30°	$\frac{\pi}{6}$	**0.5**	**0.866**	**0.577**
45°	$\frac{\pi}{4}$	**0.707**	**0.707**	**1**
60°	$\frac{\pi}{3}$	**0.866**	**0.5**	**1.732**
90°	$\frac{\pi}{2}$	**1**	**0**	**Undefined**
120°	$\frac{2\pi}{3}$	**0.866**	**-0.5**	**1.732**
135°	$\frac{3\pi}{4}$	**0.707**	**-0.707**	**-1**
150°	$\frac{5\pi}{6}$	**0.5**	**-0.866**	**-0.577**
180°	π	**0**	**-1**	**0**
210°	$\frac{7\pi}{6}$	**-0.5**	**-0.866**	**0.577**
225°	$\frac{5\pi}{4}$	**-0.707**	**-0.707**	**1**
240°	$\frac{4\pi}{3}$	**-0.866**	**-0.5**	**1.732**
270°	$\frac{3\pi}{2}$	**-1**	**0**	**Undefined**

300°	$\frac{5\pi}{3}$	-0.866	0.5	-1.732
315°	$\frac{7\pi}{4}$	-0.707	0.707	-1
330°	$\frac{11\pi}{6}$	-0.5	0.866	-0.577
360°	2π	0	1	0

Teacher Tip: The questions in the document are starting points for discussions about the patterns in the values of trig functions. Teachers should be prepared to discuss patterns beyond the ones in the document.

Problem 2 – Searching for Patterns

Use the values in the table to respond to the following questions.

1. Find the values of θ where $\sin\theta$ is positive.

   Solution: $0° < \theta < 180°$ *or* $0 < \theta < \pi$

2. Find the values of θ where $\cos\theta$ is negative.

   Solution: $90° < \theta < 270°$ *or* $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$

3. Find the values of θ where $\tan\theta$ is positive. Find the values of θ where $\tan\theta$ is negative. Explain.

   Solution: Positive: $0° < \theta < 90°$ *and* $180° < \theta < 270°$ because sin and cos have the same sign
   Negative: $90° < \theta < 180°$ *and* $270° < \theta < 360°$ because sin and cos have different signs

4. Find the angle θ where $\cos\theta = \cos 30°$.

   Solution: $\cos 330°$

5. Name two other pairs of angles where the cosine of the angle is the same.

   Possible solution: $\cos 45° = \cos 315°$, $\cos 60° = \cos 300°$

6. Find the angle θ where $\tan\theta = \tan 45°$.

Solution: $\tan 225°$

7. Name two other pairs of angles where the tangent of the angle is the same.

   Possible solution: $\tan 60° = \tan 240°$, $\tan 30° = \tan 210°$

8. Record all the patterns you see with the sine function.

   Possible solution: Answers will vary. Sample: The values in the first quadrant are repeated in the other quadrants, but have different signs.

9. Describe any other patterns you see.

   Possible solution: Answers will vary. Sample: The values of sine and cosine switch within a quadrant, such as $\sin 30° = \cos 60°$, but have different signs.

10. Describe what happens at $0°, 90°, 180°, 270°, and\ 360°$.

    Possible discussions: These are the angles that are farthest from the center of the unit circle, and have x and y coordinates of either 0 or 1. These values result in trig ratios that equate to 0, 1, or undefined. Also, $0°\ and\ 360°$ have the same results as they share the same x- and y-coordinates.

11. Explain why the tangent function is undefined for some angle measures.

    Possible explanation: Since $\tan\theta = \frac{y}{x}$, anytime $x = 0$ along the unit circle, tangent will be undefined.

Teacher Guidance: In this next problem, students will repeat the activity for the reciprocal trigonometric functions. Students should notice that these functions are reciprocals of the functions from the first part of the activity by looking at the given formulas. This will help them is calculating these functions because they can simply find the reciprocals on a Calculator page or the Scratchpad instead of finding new ratios.

Problem 3 – Patterns in Reciprocal Functions

Using the unit circle, the reciprocal trig functions can be defined as follows:

$\csc\theta = \frac{h}{y}$ $\sec\theta = \frac{h}{x}$ $\cot\theta = \frac{x}{y}$

Complete the following table by finding the reciprocals from the computed values on the first table.

θ	$\csc\theta$	$\sec\theta$	$\cot\theta$
0°	**Undefined**	**1**	**Undefined**
30°	**2**	**1.155**	**1.732**
45°	**1.414**	**1.414**	**1**
60°	**1.155**	**2**	**0.577**
90°	**1**	**Undefined**	**0**
120°	**1.155**	**-2**	**-1.732**
135°	**1.414**	**-1.414**	**-1**
150°	**2**	**-1.155**	**-1.732**
180°	**Undefined**	**-1**	**Undefined**
210°	**-2**	**-1.155**	**1.732**
225°	**-1.414**	**-1.414**	**1**
240°	**-1.155**	**-2**	**0.577**
270°	**-1**	**Undefined**	**0**
300°	**-1.155**	**2**	**-0.577**
315°	**-1.414**	**1.414**	**-1**
330°	**-2**	**1.144**	**-1.732**
360°	**Undefined**	**1**	**Undefined**

Use the values in the table to respond to the following questions.

1. Record any patterns that you see.

   Possible solution: The same patterns that appeared in the first table are showing up in this table, but now instead of sin forming patterns with cos, csc is forming patterns with sec. Csc is positive and negative in the same quadrants as sin was positive and negative.

2. Discuss with a classmate if you notice if any of the functions are undefined. Find which functions and for what values of θ they are undefined.

Possible discussion: Since these three functions are the reciprocals of sin, cos and tan, and since x and y are, at times, zero, this will result in having zero in the denominator and therefore causing each of the three reciprocal trig functions (csc, sec, and cot) to be undefined at either $0°, 90°, 180°, 270°\ or\ 360°$.

Further IB Application

In this application, students should use one of the following trig identities and the information used in the previous three problems to answer the questions (a) and (b):

$$\mathbf{\cos 2\theta = cos^2\theta - sin^2\theta = 2cos^2\theta - 1 = 1 - 2sin^2\theta}$$

(a) Show that the equation $\cos 2\theta = \cos\theta$ can be written in the form $2\cos^2\theta - \cos\theta - 1 = 0$.

Solution: Replace $\cos 2\theta$ with its identity $2\cos^2\theta - 1$

$2\cos^2\theta - 1 = \cos\theta$

Set this equation = 0

$2cos^2\theta - \cos\theta - 1 = 0$

(b) Hence, solve $\cos 2\theta = \cos\theta$ where $0 \leq \theta \leq \pi$.

Solution: Factor the result from part (a)

$(2\cos\theta + 1)(\cos\theta - 1) = 0$

Solve each factor by setting each = 0

$$2\cos\theta + 1 = 0 \quad and \quad \cos\theta - 1 = 0$$
$$\cos\theta = -\frac{1}{2} \quad and \quad \cos\theta = 1$$
$$\theta = 120°\ or\ \frac{2\pi}{3} \text{ and } \theta = 0°\ or\ 0$$

Teacher Tip: Throughout this activity, the students are asked to discuss with classmates and explain how they achieved their answers. This is a wonderful opportunity to create a student led classroom. As you float around the room, listen to what they are saying, add to their discussions, and give them leading questions to see how they respond.

TI-Nspire Navigator Opportunity: ***Quick Poll (Open Response)***
Any part to any Problem in the activity would be a great way to quickly assess your student's understanding of finding and discussing both forms of Scientific Notation and Expanded Form.

***Note: This activity has been developed independently by Texas Instruments and aligned with the IB Mathematics curriculum, but is not endorsed by IB™. IB is a registered trademark owned by the International Baccalaureate Organization.*